Efficiency Enhancement of Probabilistic Model Building Genetic Algorithms

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Motivation

- Probabilistic model building genetic algorithms:
  - Solve hard problems quickly, reliably, and accurately
  - Intractability → tractability
  - Polynomial (usually subquadratic) scalability
    - Daunting for complex problems
    - 1000 variables, 10 seconds/evaluation → ~120 days!
- Efficiency-Enhancement Techniques
  - Tractability → Practicality
- Evaluation relaxation via endogenous fitness-estimation model
- Local search in linkage neighborhoods

Outline

- Background and purpose
- Methodology
- Evaluation relaxation using an internal probabilistic fitness-estimation model
  - Methodology
  - Scalability and speed-up analysis
- Incorporation of a competent mutation operator
  - Methodology
  - Scalability and speed-up analysis
- Future work, summary and conclusions

Background and Purpose

- Efficiency Enhancement of GAs
  - Parallelization [Cantu-Paz, 2001]
  - Time utilization [Goldberg, 1999; Goldberg & Srivastava, 2001]
  - Hybridization [Davis, 1990; Goldberg & Voessner, 1999]
  - Evaluation relaxation [Sastry, 2002]
- Solve boundedly difficult problems in polynomial time
  - Propose two principled efficiency-enhancement techniques
- Use probabilistic models built by PMBGAs
  - Develop and use an inexpensive fitness-estimation model
  - Local search in the linkage neighborhood

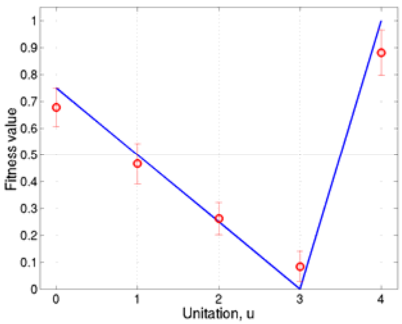
Evaluation Relaxation in PMBGAs

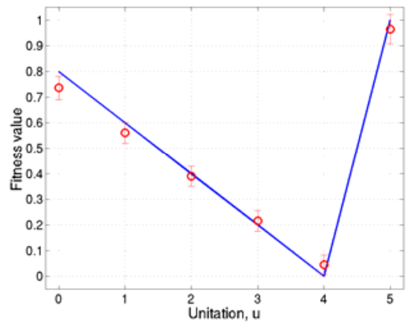
- Replace accurate, but expensive fitness evaluation with less-accurate, but inexpensive fitness estimation
  - Fitness inheritance [Smith *et al*, 1994; Sastry *et al*, 2000]
  - Simple method yields limited speed-up of 1.25
- Develop and use an internal probabilistic fitness model
  - Automatic discovery of fitness structure
  - Estimate sub-structure (building block) fitnesses
  - Individual fitness a function of sub-structural fitness
- Illustrate using extended compact genetic algorithm (eCGA)

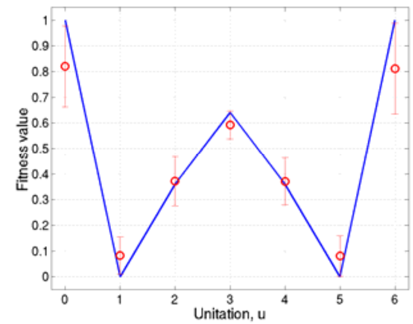
Internal Sub-Structural Fitness Model

- Identify key sub-structures of the search problem
- Estimate the fitness of sub-structure instances

Fitness of substructure instance **=** Average fitness of all evaluated individuals with the schema instance **−** Average fitness of all evaluated individuals in the population







- Individual fitness as a function of sub-structural fitnesses
  - Sum of fitness estimates of sub-structure instances
  - Can use other complex methods such as neural networks

Illustration Of Sub-Structural Fitness Estimation

- Use model-building procedure of extended compact GA
  - Partition genes into (mutually) independent groups

Model Structure	Metric
$[X_0]$ $[X_1]$ $[X_2]$ $[X_3]$ $[X_4]$ $[X_5]$ $[X_6]$ $[X_7]$ $[X_8]$ $[X_9]$ $[X_{10}]$ $[X_{11}]$	1.0000
$[X_0]$ $[X_1]$ $[X_2]$ $[X_3]$ $[X_4X_5]$ $[X_6]$ $[X_7]$ $[X_8]$ $[X_9]$ $[X_{10}]$ $[X_{11}]$	0.9933
$[X_0]$ $[X_1]$ $[X_2]$ $[X_3]$ $[X_4X_5X_7]$ $[X_6]$ $[X_8]$ $[X_9]$ $[X_{10}]$ $[X_{11}]$	0.9819
$[X_0]$ $[X_1]$ $[X_2]$ $[X_3]$ $[X_4X_5X_6X_7]$ $[X_8]$ $[X_9]$ $[X_{10}]$ $[X_{11}]$	0.9644
⋮	⋮
$[X_0]$ $[X_1]$ $[X_2]$ $[X_3]$ $[X_4X_5X_6X_7]$ $[X_8X_9X_{10}X_{11}]$	0.9273
⋮	⋮
$[X_0X_1X_2X_3]$ $[X_4X_5X_6X_7]$ $[X_8X_9X_{10}X_{11}]$	0.8895

- Use population data to estimate:
  - Probabilities: $p(X_0X_1X_2X_3)$, $p(X_4X_5X_6X_7)$, $p(X_8X_9X_{10}X_{11})$
  - Sub-structure fitnesses: $f(X_0X_1X_2X_3)$, $f(X_4X_5X_6X_7)$, $f(X_8X_9X_{10}X_{11})$

Scalability of Evaluation-Relaxation Scheme

- Error in fitness estimation modeled as additive Gaussian noise

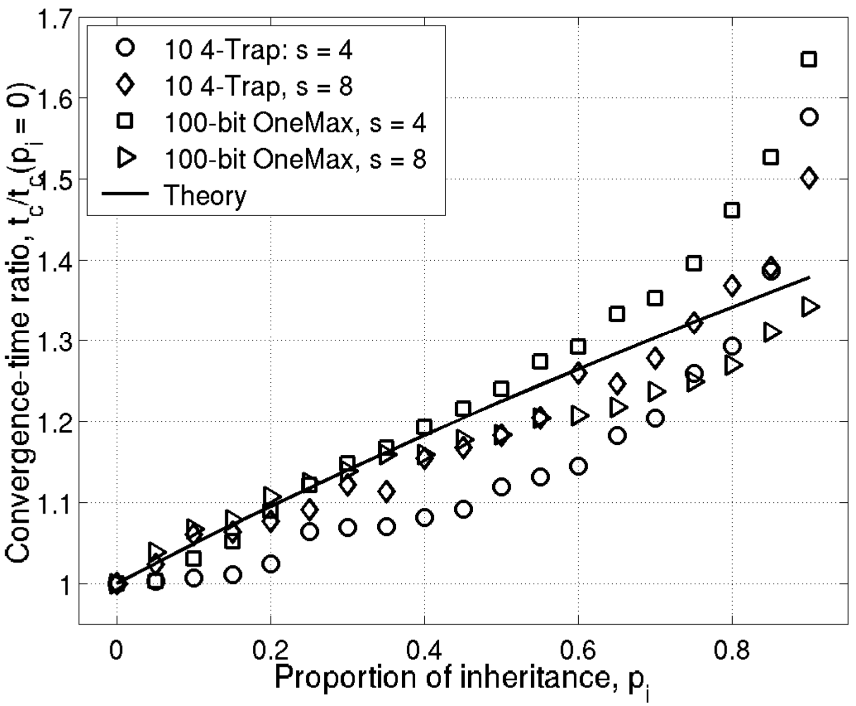
$$\mu_{\epsilon,t} = 0, \quad \sigma^2_{\epsilon,t} = p_i \sigma^2_{f,t}$$

- Convergence time: Selection-Intensity based models

[Mühlenbein & Schlierkamp-Voosen, 1993; Thierens & Goldberg 1993; Bäck, 1994; Miller & Goldberg, 1995 & 1996; Sastry & Goldberg, 2002]

$$t_{c,r} = \frac{t_c}{t_{c,o}} \approx \sqrt{1 + p_i}$$

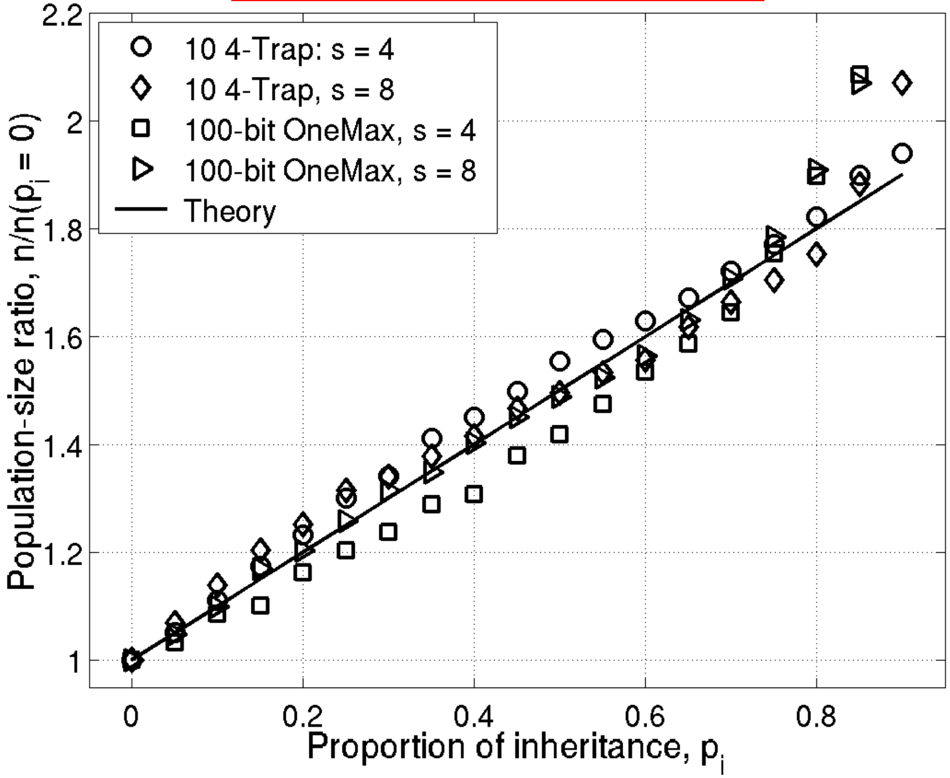
- Neglected cumulative effects of fitness estimation



Scalability of Evaluation-Relaxation Scheme

- Population Sizing Model [Goldberg *et al*, 1992; Harik *et al*, 1997; Pelikan *et al*, 2001]
  - BB supply,
  - Decision-making, and
  - Accurate model building

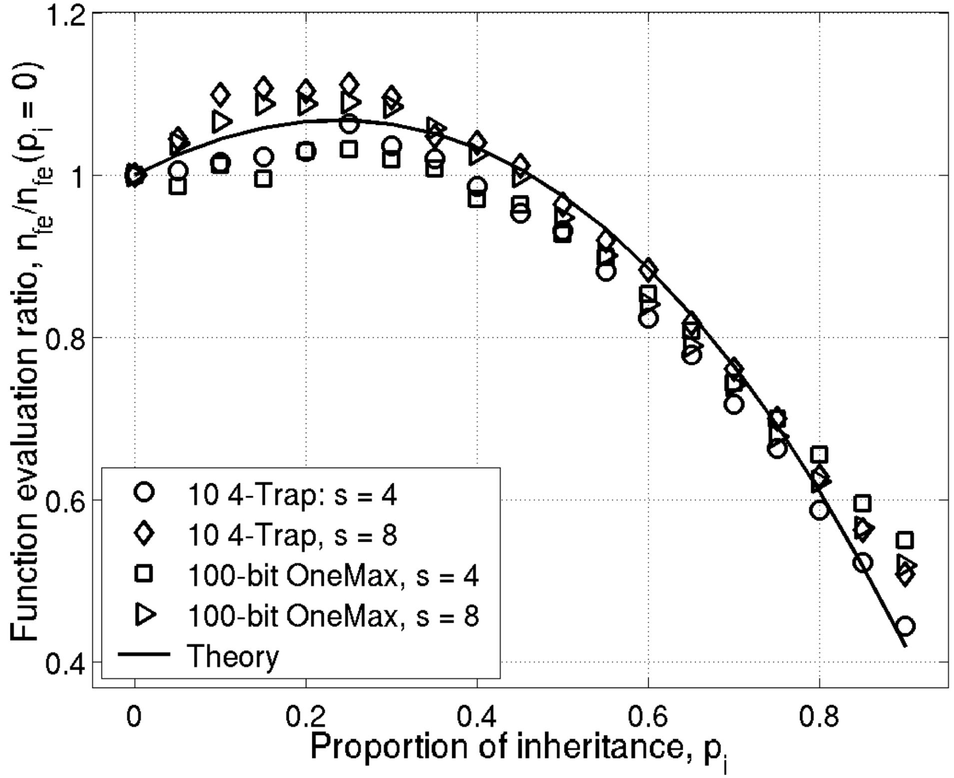
$$n_r = \frac{n}{n_o} \approx 1 + p_i$$



Scalability Of Evaluation-Relaxation Scheme

- **Scalability:** # function evaluations required to obtain a solution of quality 1-1/m
  - $1-p_i$ individuals evaluated
  - Neglected the cost of building and using the fitness-estimation model

$$n_{fe,r} \approx (1-p_i)(1+p_i)^{1.5}$$



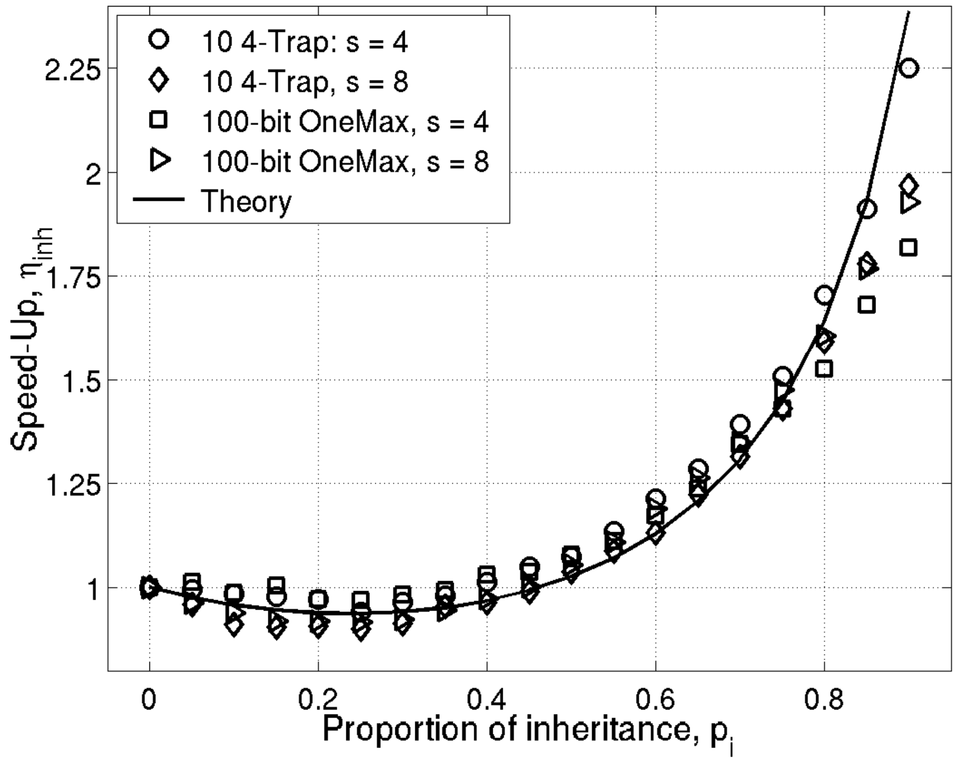
Scalability Of Evaluation-Relaxation Scheme

- Speed-Up: Ratio of # function evaluations with evaluation relaxation to that without it

- Only 1-15% individuals need evaluation
  - Optimal $p_i \sim 0.85$-0.99

- Speed-Up: 1.75-53

$$\eta \approx (1 - p_i)^{-1} (1 + p_i)^{-1.5}$$

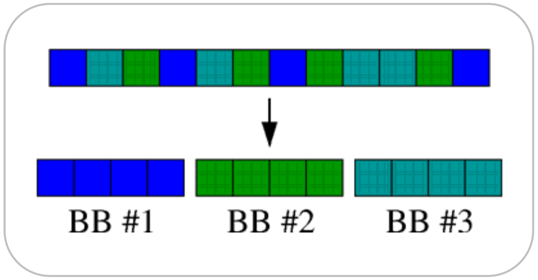


Designing A Competent Mutation

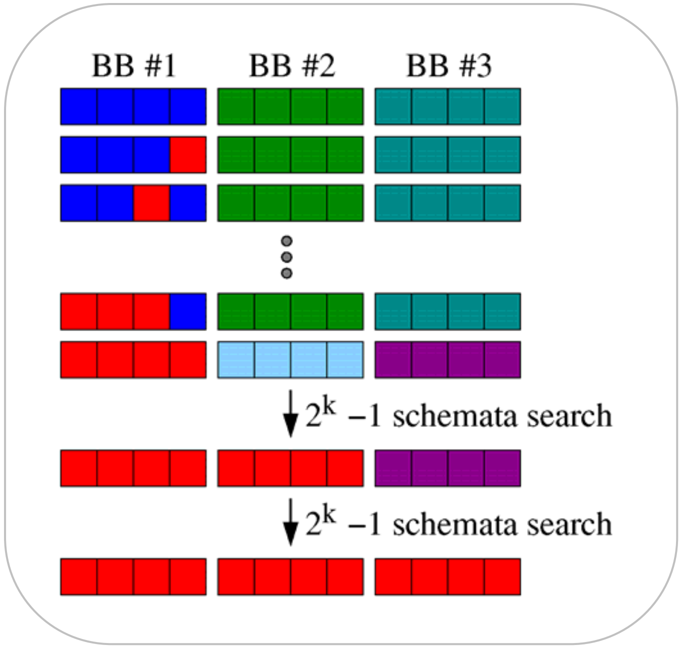
- Systematic induction of good neighborhood information
  - Based on a population of promising candidate solutions
- Polynomial (quadratic) scalability
- Perform scalability analysis of the proposed algorithm
  - Derive and use *facetwise* models
  - Verify on nearly decomposable problems of bounded difficulty
    - Additively separable problems
  - Compare with a competent selectorecombinative GA

Designing A Competent Mutation Operator

- Inducing good neighborhoods
  - Sample of candidate solutions
  - Use linkage-learning procedures developed for selectorecombinative GAs



- Neighborhood: Space vs. Sub-space
- Search: Hillclimbing vs. Random
  - Consider linkage partitions
    - Arbitrary left-to-right order
  - Choose the best schemata
    - Among the 2^k possible ones



How to Induce Neighborhoods?

- Induce good neighborhoods as linkage groups
- Use model-building procedure of extended compact GA
  - Partition genes into (mutually) independent groups
  - Start with the lowest complexity model
  - Search for a least-complex, most-accurate model

Model Structure	Metric
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Algorithmic Description of Selectomutative GA

1. Initialize the population (usually random initialization)

2. Evaluate the fitness of individuals

3. Select promising solutions (e.g., tournament selection)

4. Build the probabilistic model
   - Optimize the structure and parameter of the probabilistic model to best fit the selected individuals
   - Automatic discovery linkage neighborhoods

5. Apply enumerative BB-wise mutation
   - Search for the best building block in each partition

Scalability of Competent Mutation Operator

- Population size required for accurate modeling of neighborhood information [Pelikan, Sastry, and Goldberg, 2002]:

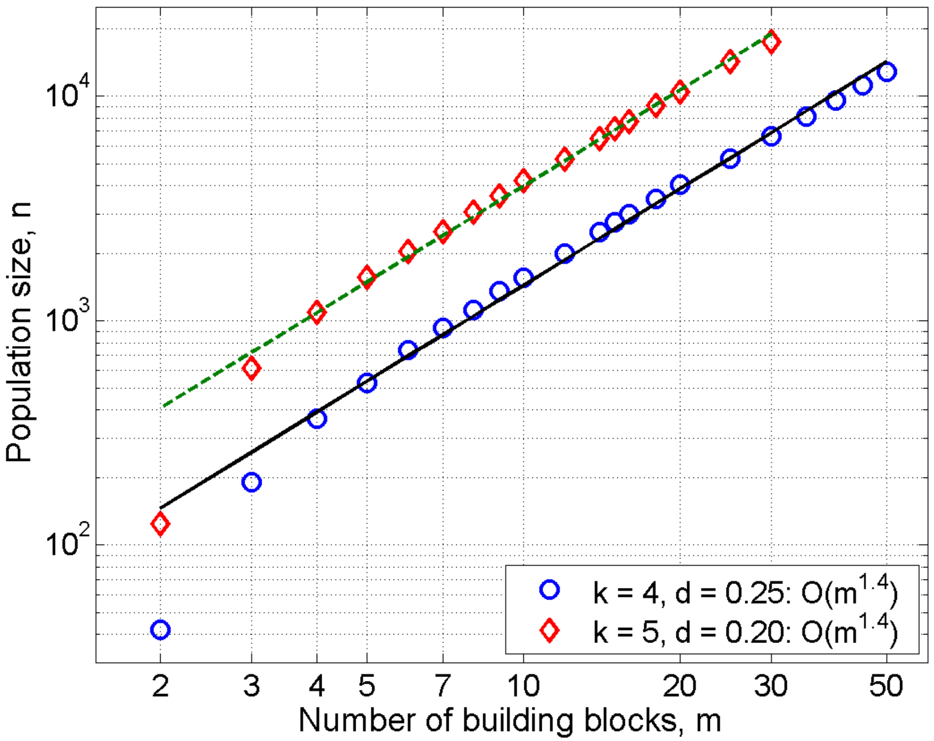
$$\mathcal{O}\left(2^k \cdot m^{1.05}\right) \leq n \leq \mathcal{O}\left(2^k \cdot m^{2.1}\right)$$

- Empirical verification:

$$n = \mathcal{O}\left(2^k \cdot m^{1.5}\right)$$

Sub-Component size (BB size)

Components (# BBs)



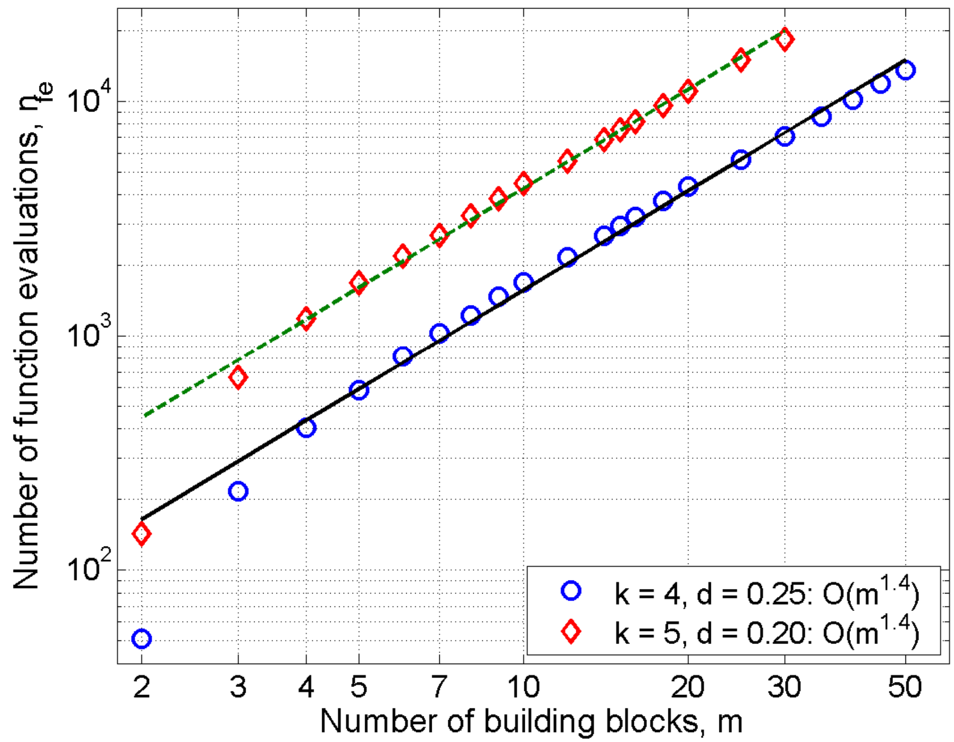
Scalability of Competent Mutation Operator

- Model building cost: $$\mathcal{O}\left(2^k \cdot m^{1.05}\right) \leq n_{fe} \leq \mathcal{O}\left(2^k \cdot m^{2.1}\right)$$

- BB-wise enumeration cost: $$n_{fe} = \left(2^k - 1\right) m = \mathcal{O}\left(2^k \cdot m\right)$$

- Empirical verification

$$n_{fe,M} = \mathcal{O}\left(2^k \cdot m^{1.5}\right)$$



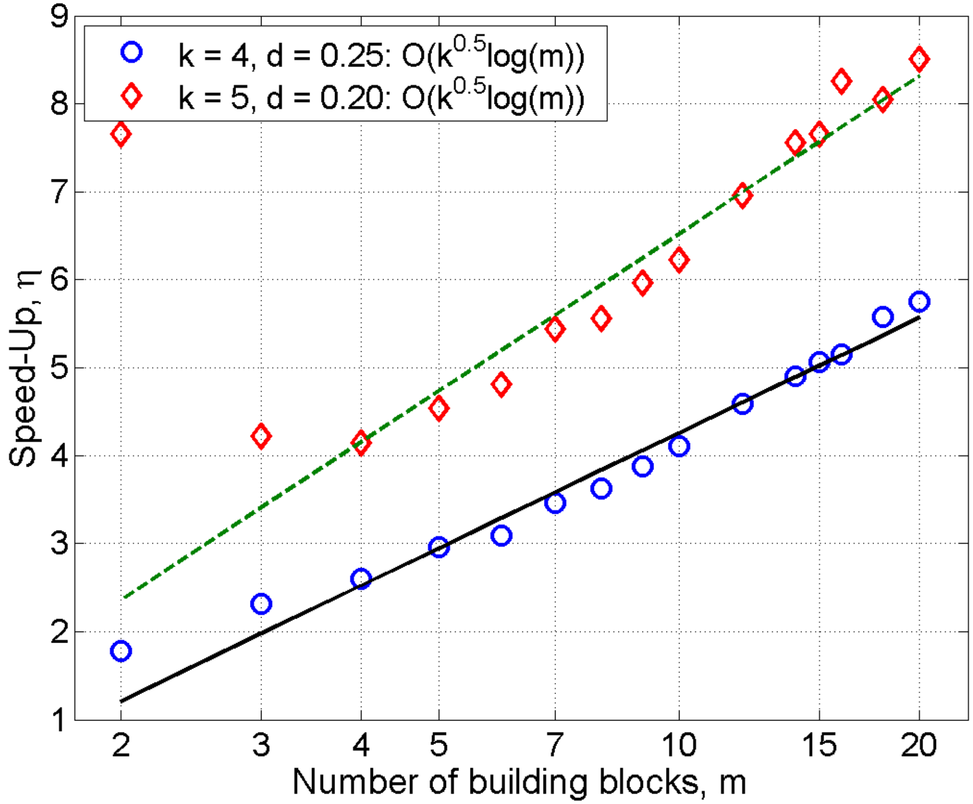
Competent Crossover vs. Competent Mutation

Competent mutation:

$$\mathcal{O}\left(2^k m^{1.5}\right)$$

Competent crossover:

$$\mathcal{O}\left(2^k m^{1.5} \log m\right)$$



- Speed-Up: Scalability ratio of mutation to that of crossover

$$\eta = \frac{n_{fe,M}}{n_{fe,XO}} = \mathcal{O}\left(\sqrt{k} \log m\right)$$

Future Work

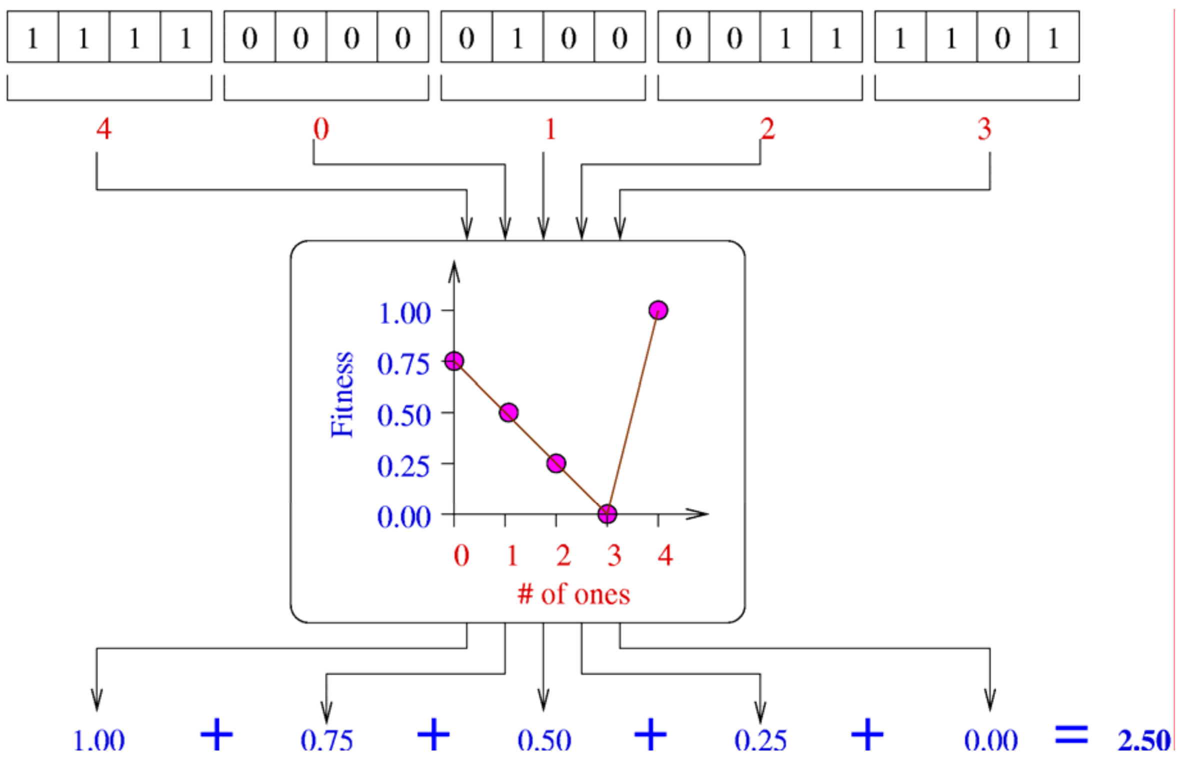
- Hybridization of competent crossover *and* mutation
  - Consider a hybrid GA with *oscillating* population
- Problems with overlapping building blocks
  - Effect is similar to that of exogenous noise
  - Speed-Up likely to be lower
  - More complex model-building schemes (E.g., BOA, DSMGA)
- Problems with non-uniform salience
  - Account for sequentiality
- Hierarchical problems
  - Account for effect of niching

Summary & Conclusions

- Evaluation relaxation using internal fitness model
  - Uses sub-structural fitness estimates as basis
  - Only 1-15% of the individuals need fitness evaluation
  - Yields a speed-up of 1.75-53
- Competent mutation operator:
  - Systematic induction of good neighborhoods
  - Neighborhood: Entire search space → Linkage subspace
  - Search: Hillclimbing → Random/Enumerative
  - Subquadratic scalability: $O(2^k m^{1.5})$
    - $e^k m^k$ scalability of simple mutation [Mühlenbein, 1991]

Test Problems: m-k Deceptive Trap Functions

- Building block identification is critical to GA success
- *m* concatenated deceptive trap functions [Ackley, 1987; Goldberg, 1987; Deb & Goldberg, 1993]



Algorithmic Description of eCGA

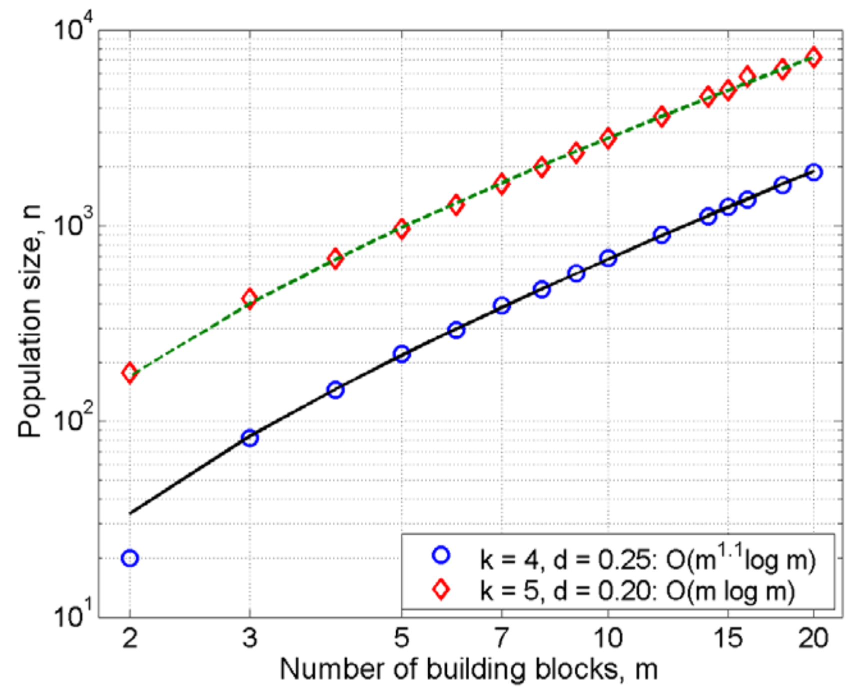
1. Initialize the population (usually random initialization)
2. Evaluate the fitness of individuals
3. Select promising solutions (e.g., tournament selection)
4. Build the probabilistic model
   - Optimize the structure and parameter of the probabilistic model to best fit the selected individuals
   - Automatic identification of building blocks
5. Sample the model to create new candidate solutions
   - Effective exchange of building blocks
6. Repeat steps 2-7 till some convergence criteria are met

Scalability of eCGA

- **Population sizing:** [Pelikan, Goldberg, Cantú-Paz, 2000; Pelikan, Sastry, & Goldberg, 2003]
  - Accounts for BB supply, decision-making, and model accuracy requirements

$$n = -\log(\alpha) \cdot \left(\frac{\sigma_{bb}}{d}\right) \cdot 2^k \cdot m$$

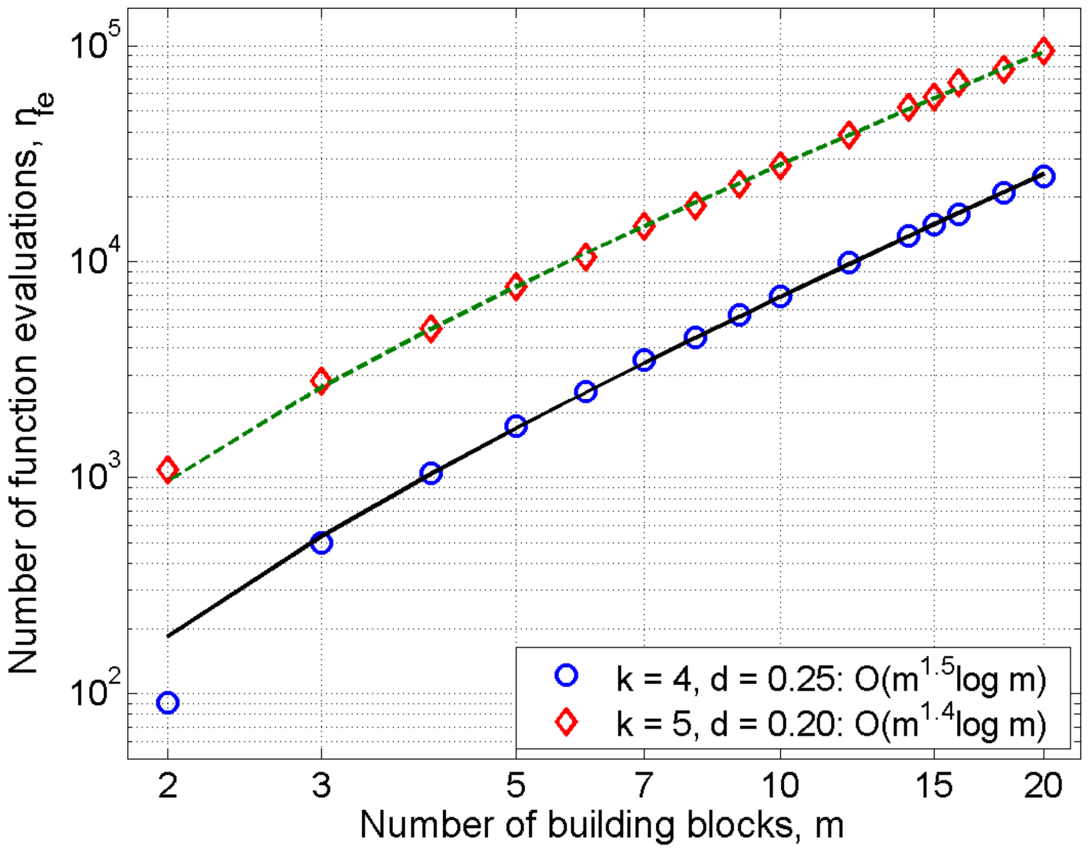
- BB fitness variance (σ_{bb})
- Signal between competing BBs (d)
- Failure probability (α)
- Sub-Component size (BB size) (k)
- Error tolerance (α)
- Signal-to-Noise ratio (σ_{bb}/d)
- # Competing sub-components (2^k)
- # Components (# BBs) (m)



Scalability of eCGA

- Number of function evaluations:
  - Error probability α = 1/m

$$
\begin{aligned}
n_{fe,XO} &\propto \left(\frac{\sigma_{bb}}{d}\right) \cdot 2^k \cdot m^{1.5} \log m \\
&= \mathcal{O}\left(2^k m^{1.5} \log m\right)
\end{aligned}
$$



Related Work

- Mutation operators in genetic algorithms
  - Random walk locally around individuals
- Mutation operators in evolution strategies [Rechenberg, 1973; Schwefel, 1977; Bäck, 1996; Beyer, 1996; Hansen & Ostermeier, 2001]
  - Neighborhood information based on entire search space
- Local search literature [Barnes *et al*, 2003; Watson, 2003; Vaughn *et al*, 2000; Armstrong & Jacobson, 2004; Glover & Laguna, 1997]
  - Emphasis on good *neighborhood* operators
  - Lack systematic methods for inducing good neighborhoods
- Mutation with known linkage neighborhoods [Sastry & Goldberg, 2004]
  - $e^k m^k$ scalability → $2^k m^2$ scalability
  - Neglected the cost of inducing linkage neighborhoods