

Scalability of Simple Genetic Algorithms: Mixing Analysis of One-Point Crossover

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ISCIS XVII

Seventeenth International Symposium
on Computer and Information Sciences

October 28–30, 2002, Orlando, FL



Foreword

- Big models
 - Accurate, high cost and opaque
- No models
 - High error, zero cost, and no guidance
- Little models: Facetwise and dimensional
 - Important middle ground
 - Quantitative guidance
 - Intuition into physical mechanism



A Simple Genetic Algorithm

- Holland, 1962; Goldberg, 1989
- Principles of natural selection and genetics
- Evolve population of candidate solutions
- Three key operators
 - **Selection:** Propagate good solutions
 - **Recombination:** Combine solutions to create new ones
 - **Mutation:** Slightly modify candidate solutions



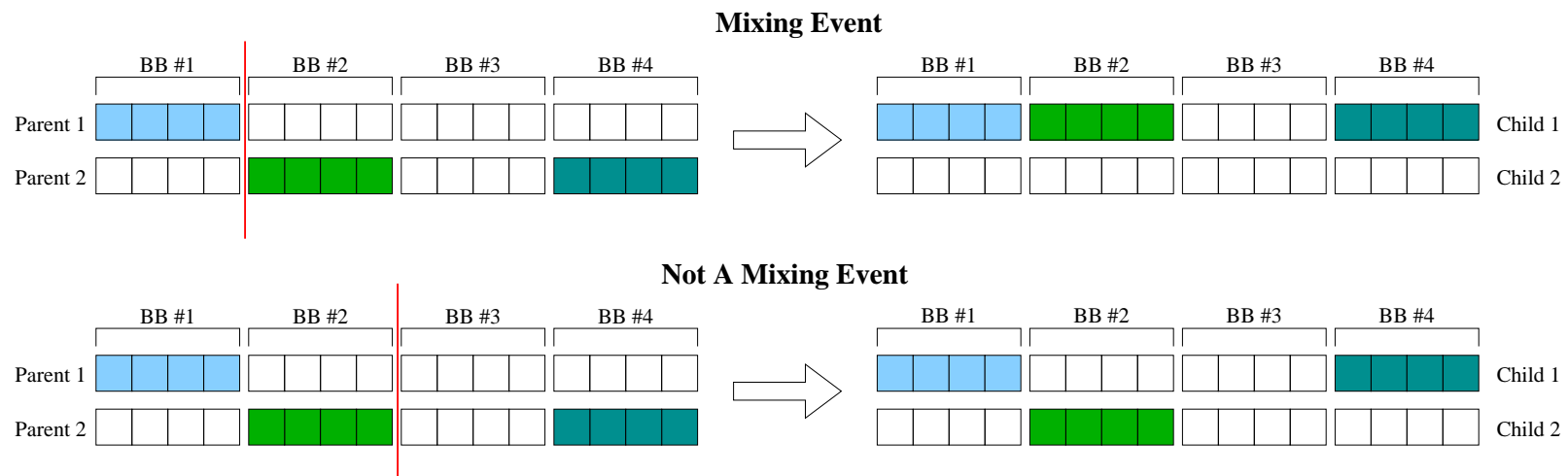
Genetic Algorithm Design Decomposition

- Goldberg (1991, 2002); Goldberg *et al* (1992)
 1. Know what GAs process—building blocks (BBs).
 2. Solve problems that are of bounded BB difficulty.
 3. Ensure an adequate supply of raw BBs.
 4. Ensure increased market share for superior BBs.
 5. Know BB takeover and convergence times.
 6. Ensure that BB decisions are well made.
 7. Ensure a good *mixing* of BBs.



What is Building-Block Mixing?

- Innovation event (Goldberg *et al*, 1993)
- Combination of BBs
 - From different parents to yield better offspring
- Crossover that leads to increase in BBs



Why Analyze Building-Block Mixing?

- BB mixing:
 - Critical for GA success.
 - Often overlooked
 - * Success with standard recombination operators
 - Need for problem-specific operators
- Facetwise models assume *tight linkage*
 - Incorporate BB mixing into GA dynamics
- Development of mixing models
 - Scalability of GAs



Overview

- Mixing problem
- Literature review
- Assumptions
- BB mixing: Worst case
- BB mixing: Best case
 - Two building-block, and m building-block case
 - Mixing-time model
 - Population-size model
- Conclusions



Mixing Problem

- Goldberg, Thierens & Deb, 1993
- How well do fixed crossover operators solve
 - GA-easy problems
 - GA-hard problems
- Mixing Time:
 - Individuals in the population have m_c BBs
 - Number of generations required to obtain individuals with $m_c + 1$ BBs



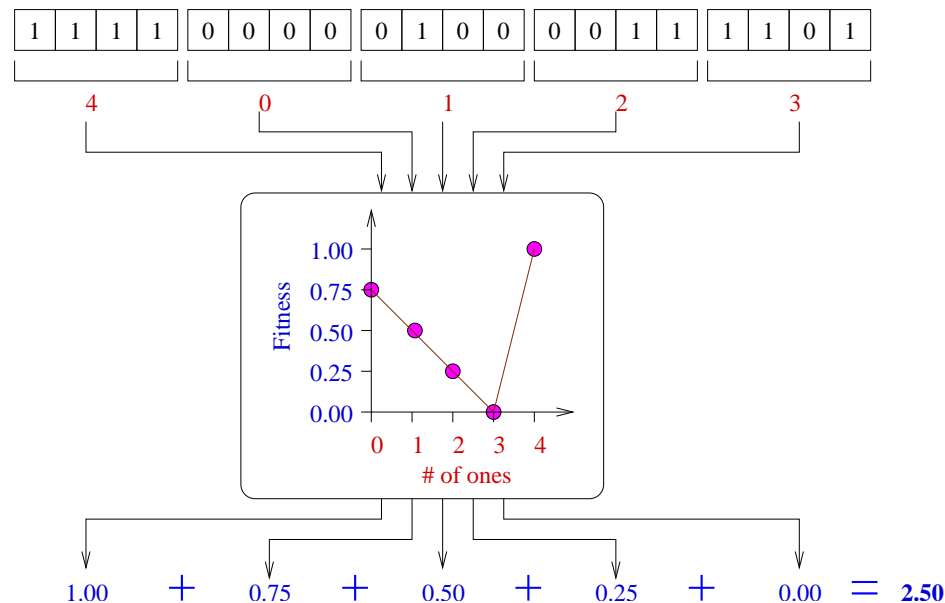
Literature Review

- Linkage equilibrium/distribution models
 - Christiansen, 1989; Booker, 1993; Rabani *et al*, 1998; Eshelman *et al*, 1989; Eshelman & Schaffer, 1995;
- Schemata evolution models
 - Holland, 1975; De Jong, 1975; Syswerda, 1989; De Jong & Spears (1990, 1991, 1992);
- All-in-one models
 - Bridges & Goldberg, 1987; Vose & Liepens, 1991; Nix & Vose, 1992; Prügel-Bennett & Shapiro, 1994;
- BB mixing models
 - Goldberg *et al*, 1993; Thierens & Goldberg, 1993; Thierens, 1995; Thierens, 1999



Assumptions

- Non-overlapping population of fixed size
- Generationwise selectorecombinative GAs
- Binary encoding and fixed string length
- Fully deceptive trap (Deb & Goldberg, 1993)



Building-Block Mixing: Worst Case

- Goldberg *et al* 1993; Thierens & Goldberg, 1993
- Uniform crossover
 - One-point crossover on loosely-linked problems

- Mixing time:

$$t_x = c \frac{2^{\mu k} 2^m}{np_c m^{5/2}}$$

- Population size: Grows exponentially

$$n \ln n > c \frac{2^{\mu k} 2^m}{np_c m^{5/2}} \ln s$$



Building-Block Mixing: Best Case

- Assumption of tight linkage
 - Alleles belonging to a BB are close to one another.
- Mixing of two BBs
- Mixing of m BBs



Building-Block Mixing: Two BBs

- At most one BB per individual ($m_c = 1$)
- Two mixing events: $\frac{b\#}{\#b}$, and $\frac{\#b}{b\#}$
- Probability of mixing

$$p_{\text{mix}} = 2 \cdot \frac{1}{2^k} \cdot \frac{2^k - 1}{2^k} \cdot \left(\frac{1}{2k - 1} \right) \approx \frac{2^{k-1}}{2k - 1}.$$

- Mixing time

$$t_x = \frac{1}{(n/2)p_c p_{\text{mix}}} = \frac{2^{k-1}(2k - 1)}{np_c}.$$



Building-Block Mixing: Two BBs

- Convergence time (Bäck, 1993)

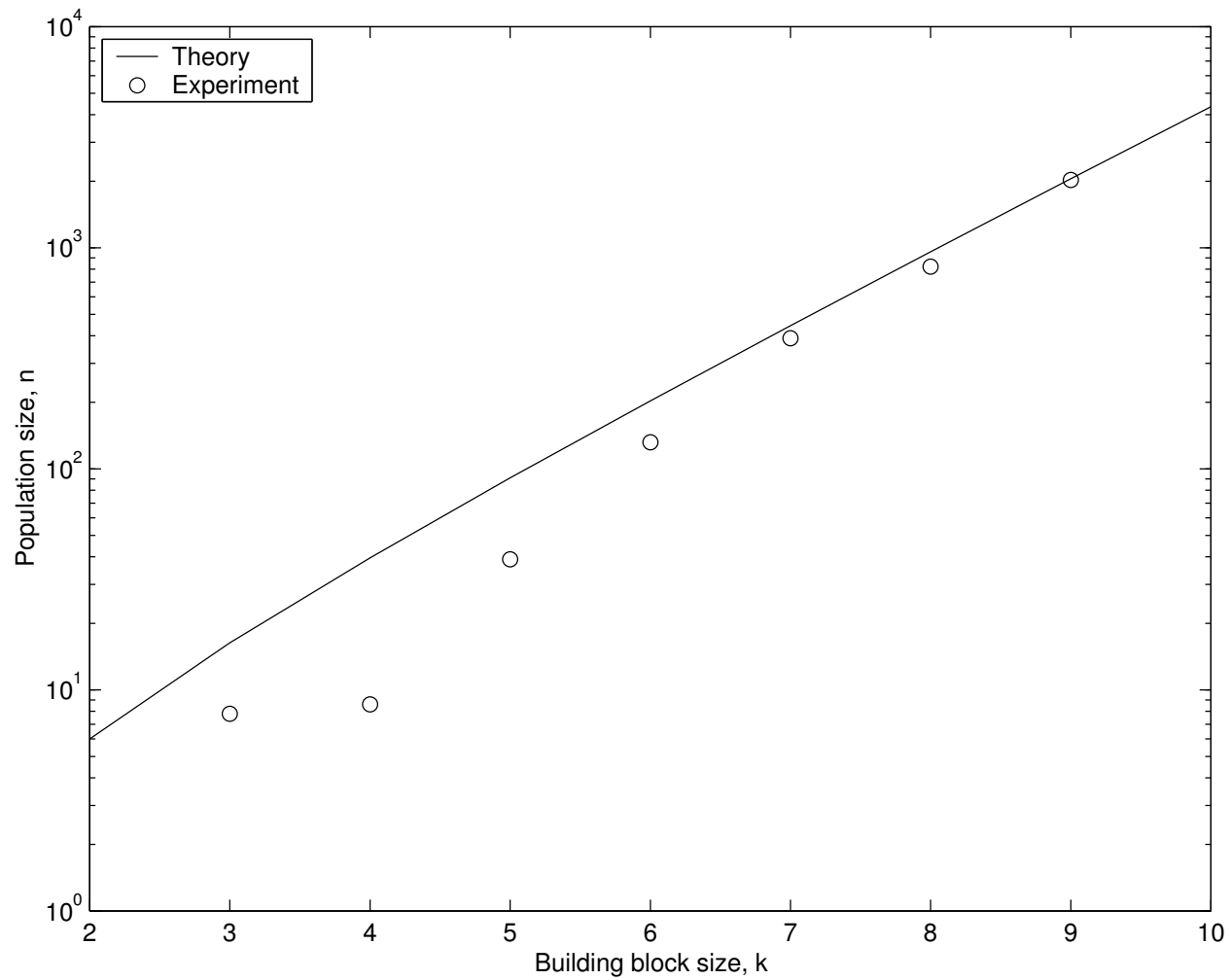
$$t_{\text{conv}} = \frac{c_c}{I} \sqrt{k \cdot m}$$

- Mixing time is greater than convergence time:
 - Premature convergence
- Mixing time is less than convergence time:

$$t_x < t_{\text{conv}} \Rightarrow n > c_x \left(\frac{I}{p_c} \right) \left(2^k \sqrt{k} \right)$$



Results: Two BB mixing

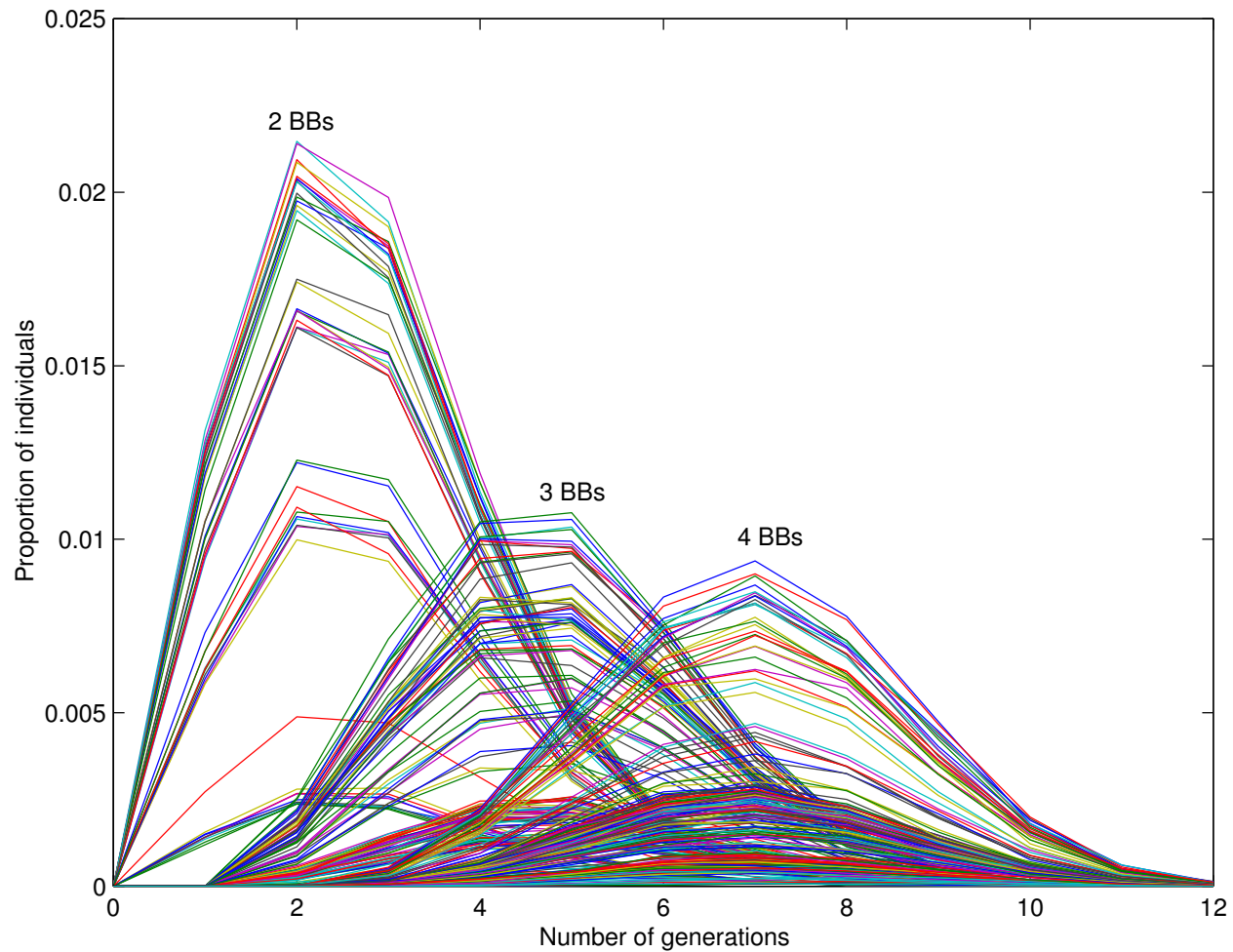


Building-Block Mixing: m BBs

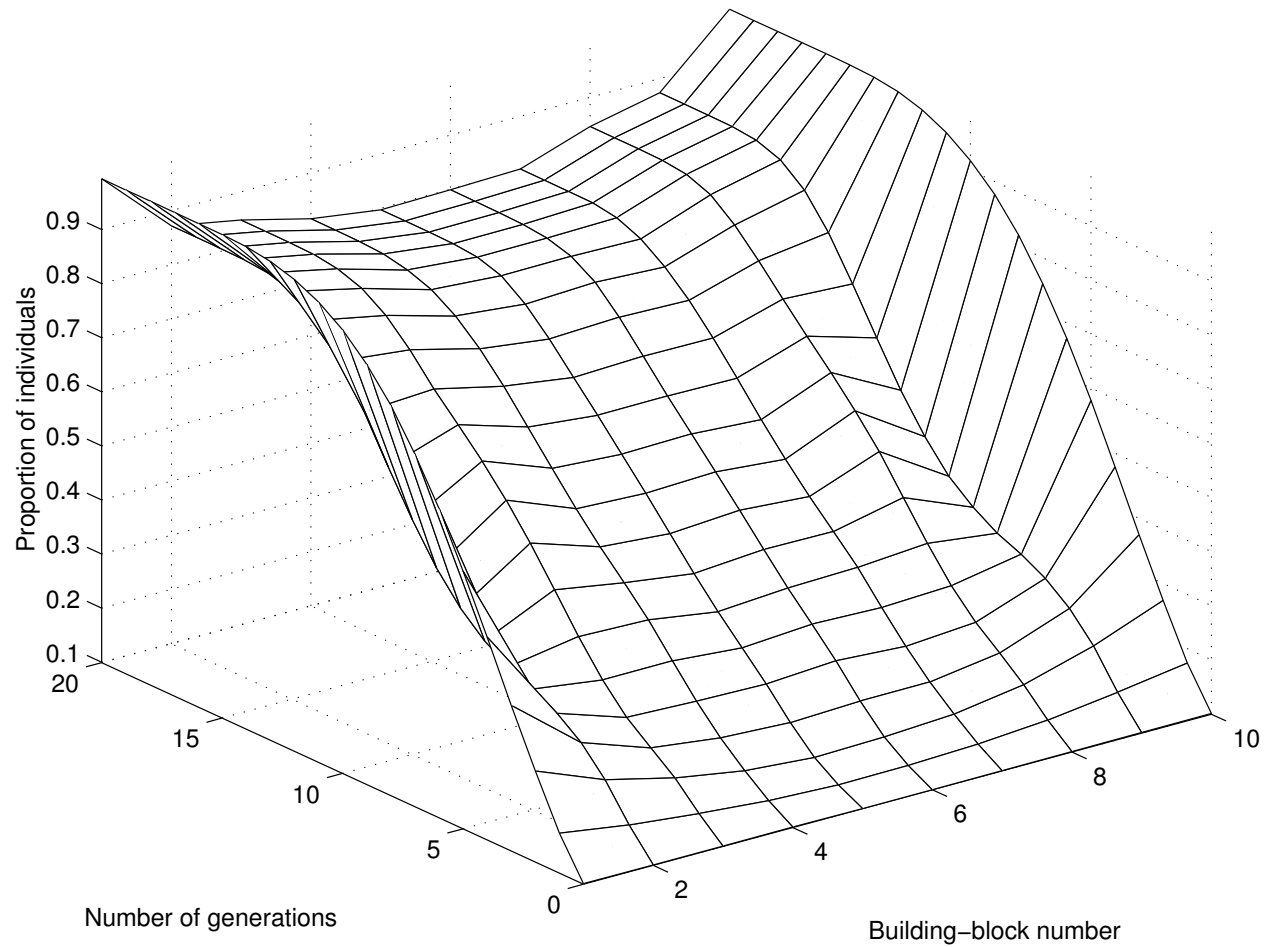
- Two key empirical observations
- *Ladder-climbing* phenomenon
 - Goldberg, Thierens & Deb (1993)
 - Mixing level increases one step at a time
- *Length-reduction* phenomenon
 - BBs at the ends get mixed often
 - BBs at string ends converge faster



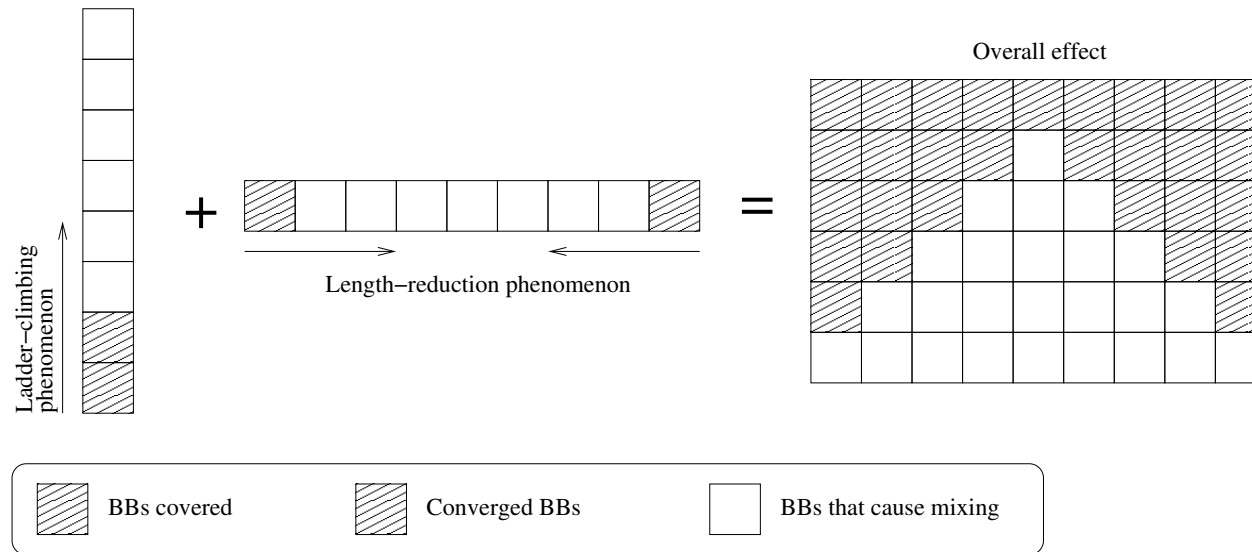
Ladder-Climbing Phenomenon



Length-Reduction Phenomenon



Ladder-Climbing + Length-Reduction



- Bounding case
 - Going from mixing level $m_c = 1$ to $m_c = 2$.



Proportion of Mixing

Recombination scenario	$b\#\#\dots\#$ $\#b\#\dots\#$	\dots	$b\#\dots\#\#\dots\#$ $\#\#\dots\#b\#\dots\#$	\dots	$b\#\dots\#$ $\#\#\dots\#$
No. of events	$2(m-1)$	\dots	$2(m-i)$	\dots	2
Mixing Prob.	$\frac{1}{mk-1}$	\dots	$\frac{(i-1)k+1}{mk-1}$	\dots	$\frac{(m-2)k+1}{mk-1}$

- Probability of mixing

$$p_{\text{mix}} \approx \frac{2}{3} \left[\frac{(m - m_c) ((m - m_c - 1)k + 3)}{(m - m_c + 1) ((m - m_c + 1)k - 1)} \right]$$

- Number of mixing events

$$n_x \approx \frac{1}{2} c_{x1} \cdot m$$



Time to Mix

- Time to climb one step of the ladder

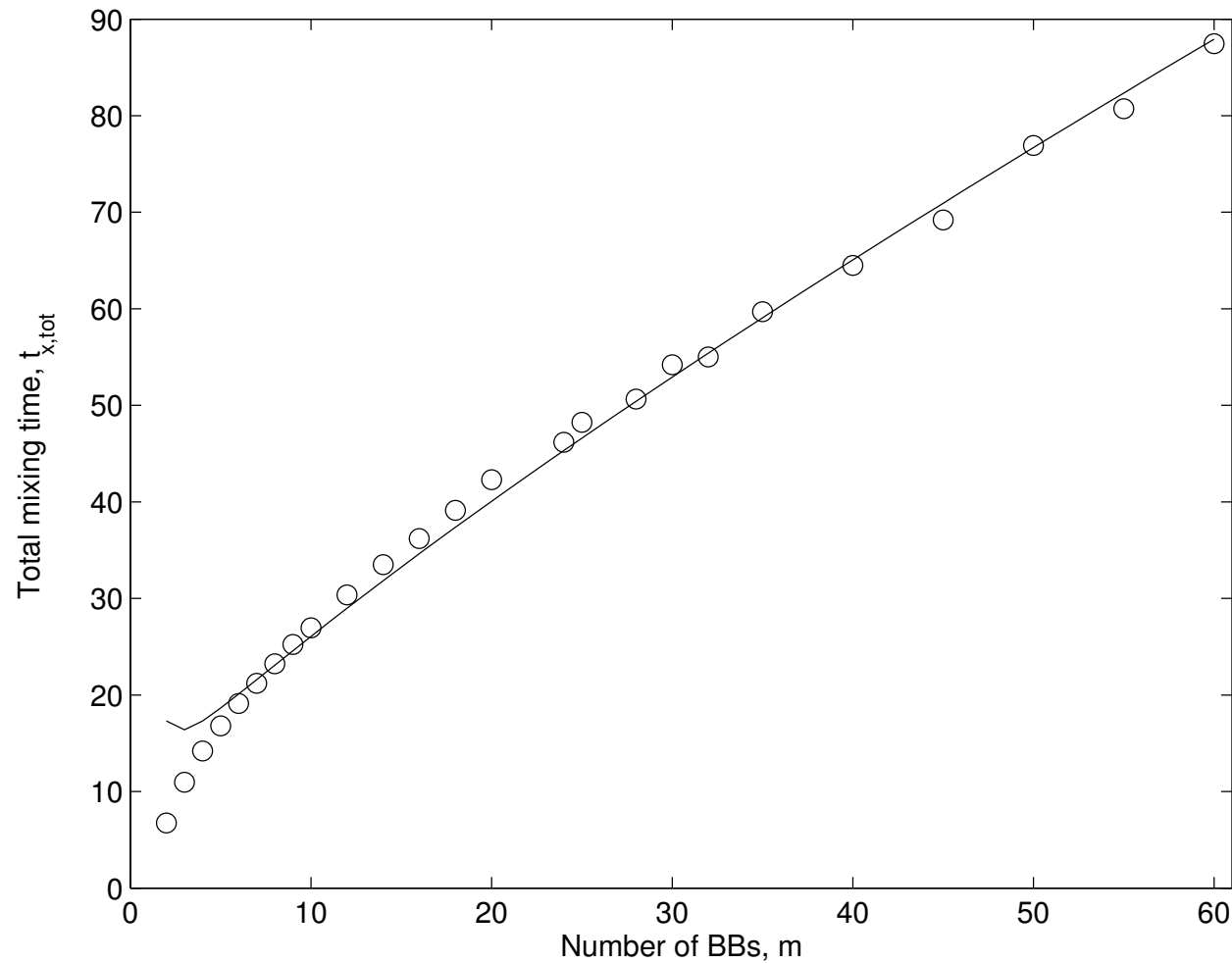
$$t_x = \frac{2^{(m-1)k} - 1}{2^{-mk}} \frac{n_x}{(n/2)p_c p_{\text{mix}}} \approx c_{x1} \frac{2^k m}{np_c}.$$

- Total mixing time
 - Time to climb all the steps of the mixing ladder

$$t_{x,\text{tot}} = m t_x = c_{x1} \frac{2^k m^2}{np_c}$$



Results: Total Mixing Time



Population-Sizing Model

- Total mixing time

$$t_{x,\text{tot}} = mt_x = c_{x1} \frac{2^k m^2}{np_c}$$

- Convergence time

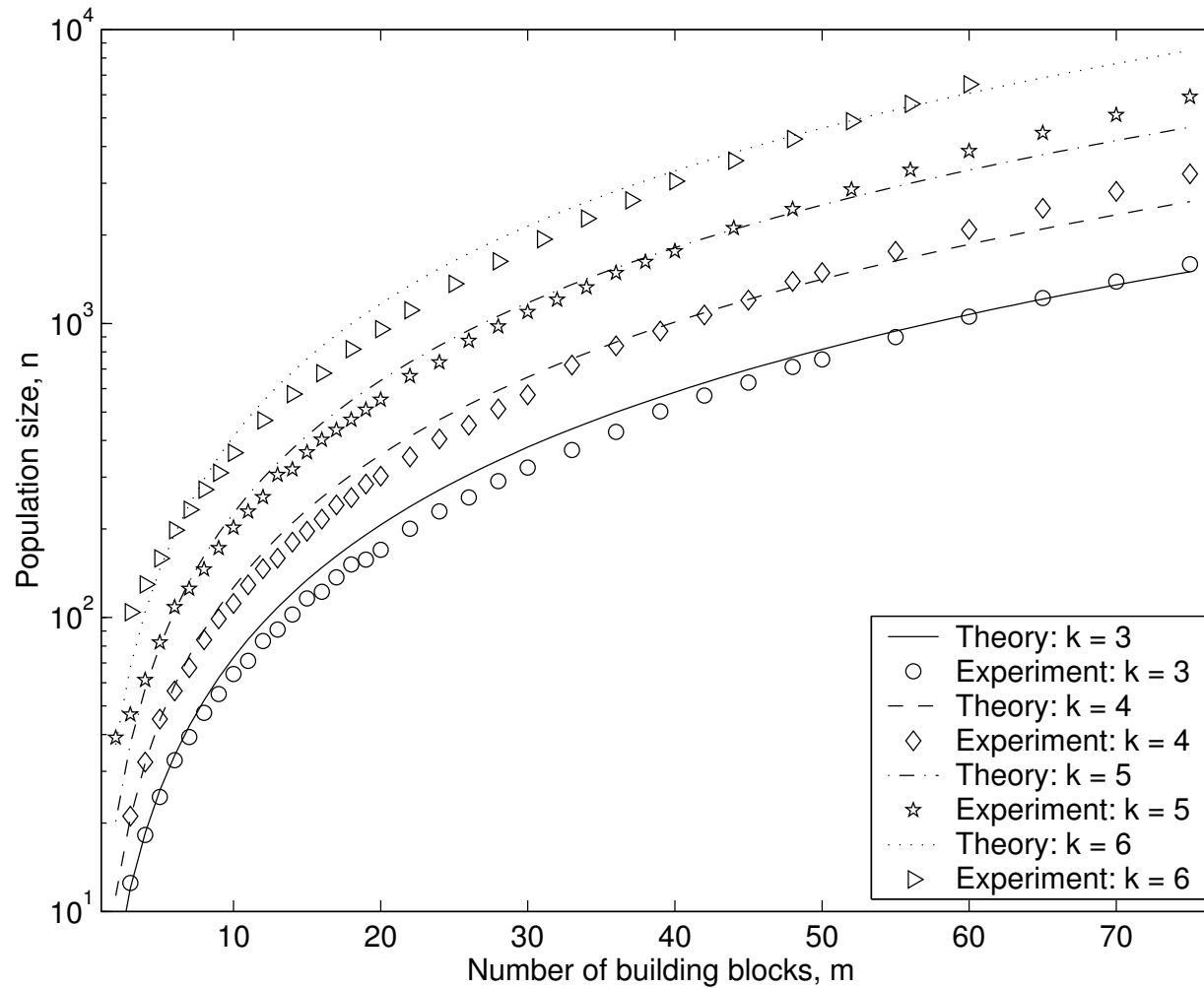
$$t_{\text{conv}} = \frac{c_c}{I} \sqrt{k \cdot m}$$

- Innovation success: $t_{x,\text{tot}} < t_{\text{conv}}$

$$n > c_{x1} \left(\frac{I}{p_c} \right) \frac{2^k m \sqrt{m}}{\sqrt{k}}$$



Results: Population Sizing



Population-Sizing Bounds

- BB decision making + BB supply
 - Goldberg *et al*, 1992; Harik *et al*, 1997;

$$n = c_n 2^k \sqrt{m}$$

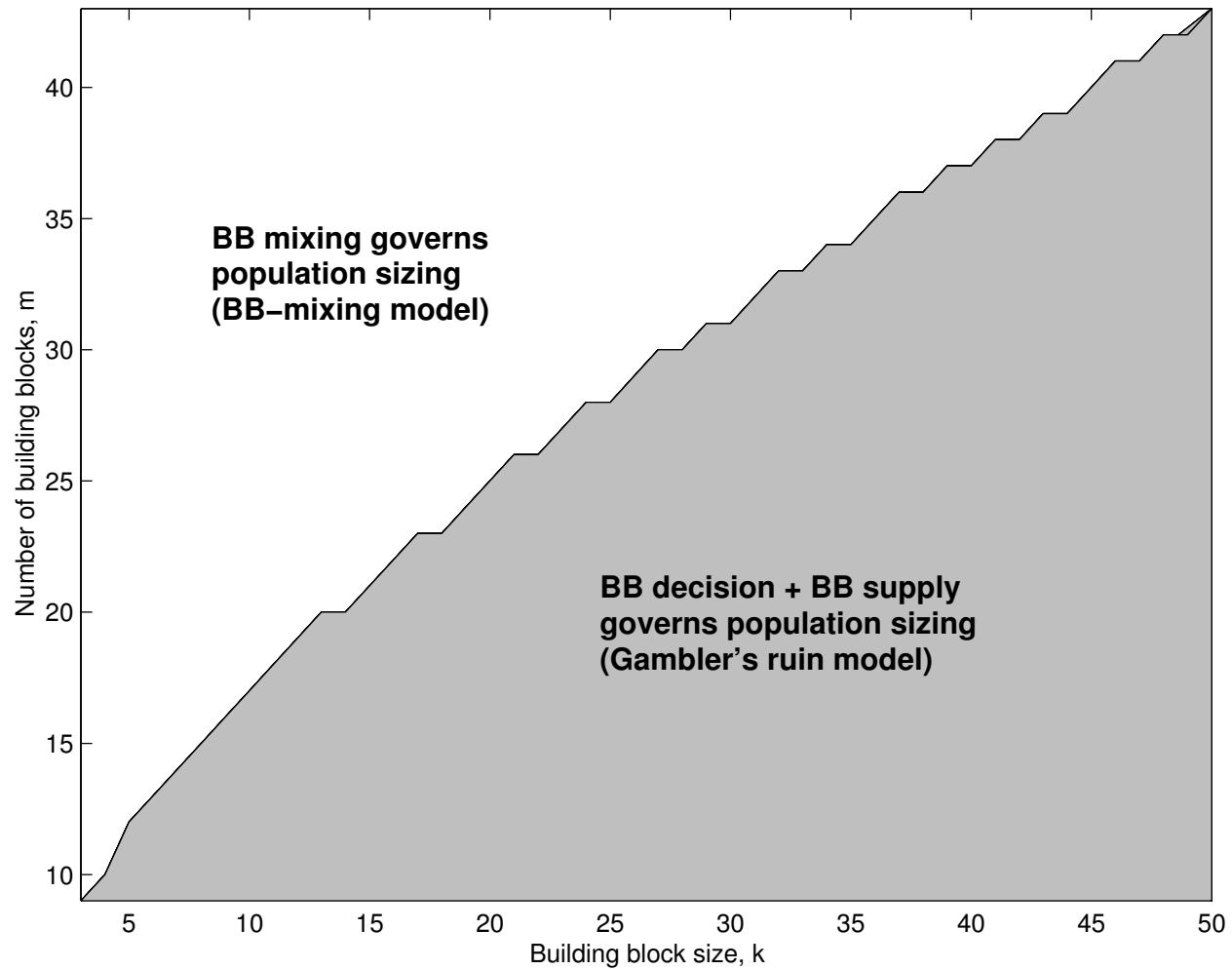
- BB mixing based model

$$n = c_{x1} \left(\frac{I}{p_c} \right) \frac{2^k m \sqrt{m}}{\sqrt{k}}$$

- BB decision making and supply governs
 - Small problems
 - problem size range increases with BB size



Population Sizing Phase Boundary



Conclusions

- BB mixing of one-point crossover
 - Tight linkage:

$$t_{x,\text{tot}} = O\left(2^k \frac{m^2}{n}\right), \quad n = O(2^k m^{1.5}).$$

- Loose linkage:

$$t_{x,\text{tot}} = n = O(2^{\mu k} 2^m).$$

- Facetwise models are useful
 - First preference in understanding complex systems
- Operators that adapt linkage are required
 - Use *competent* genetic algorithms (Goldberg, 2002)



Acknowledgments

- Air Force Office of Scientific Research, Air Force Materiel Command, USAF, under grant F49620-00-1-0163.
- National Science Foundation under grant DMI-9908252.
- Computational Science and Engineering (CSE) fellowship, University of Illinois at Urbana-Champaign.

