Modeling Tournament Selection With Replacement With Apparent Added Noise

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Foreword

- Tournament selection with replacement is noisy
- Requires more function evaluations
- Use Tournament selection **without** replacement

Overview

- Background & Motivation
- Convergence-Time Model
- Population-Sizing Model
- Modified Population-Sizing Model
- Summary

Simple Genetic Algorithms

- Based on principles of natural selection & genetics
- Evolve population of candidate solutions
- Three key operators:
  - **Selection**: Propagate good solutions
  - **Recombination**: Combine solutions to create new ones
  - **Mutation**: Minor modifications of solutions

Selection Operator

- Affects algorithm performance
  - Time to convergence
  - Solution quality
- Two classes:
  - Fitness proportionate schemes
  - Ordinal schemes
- Ordinal schemes are preferred

Tournament Selection

- Goldberg, Korb, & Deb, 1989
- Ordinal selection scheme
- s-wise tournament
  - Select s individuals randomly
  - The best among s individual wins
  - Repeat till we have n winners
- Selection of individuals:
  - With Replacement
  - Without Replacement

Motivation

- Tournament selection without replacement (TWOR):
  - Has received considerable analytical attention
  - Quality-duration models exist
- Tournament selection with replacement (TWR):
  - Usually considered equivalent to TWOR
  - This is not quite true
  - Requires separate scrutiny

Objective

- Compare TWR and TWOR
  - Convergence time
  - Population size required to achieve desired solution
- Modify existing models
  - To account for any discrepancies

Convergence Time Model

- Mühlenbein & Schlierkamp-Voosen (1993); Thierens & Goldberg (1993); Bäck (1994); Miller & Goldberg (1996)
  - Use Selection Intensity (Bulmer,1980)
- Convergence time

$$t_{\text{conv}} = \frac{\pi\sqrt{\ell}}{2I}$$

- Blickle & Thiele, 1995

$$I \approx \sqrt{2\left(\log(s) - \log\left(\sqrt{4.14\log(s)}\right)\right)}$$

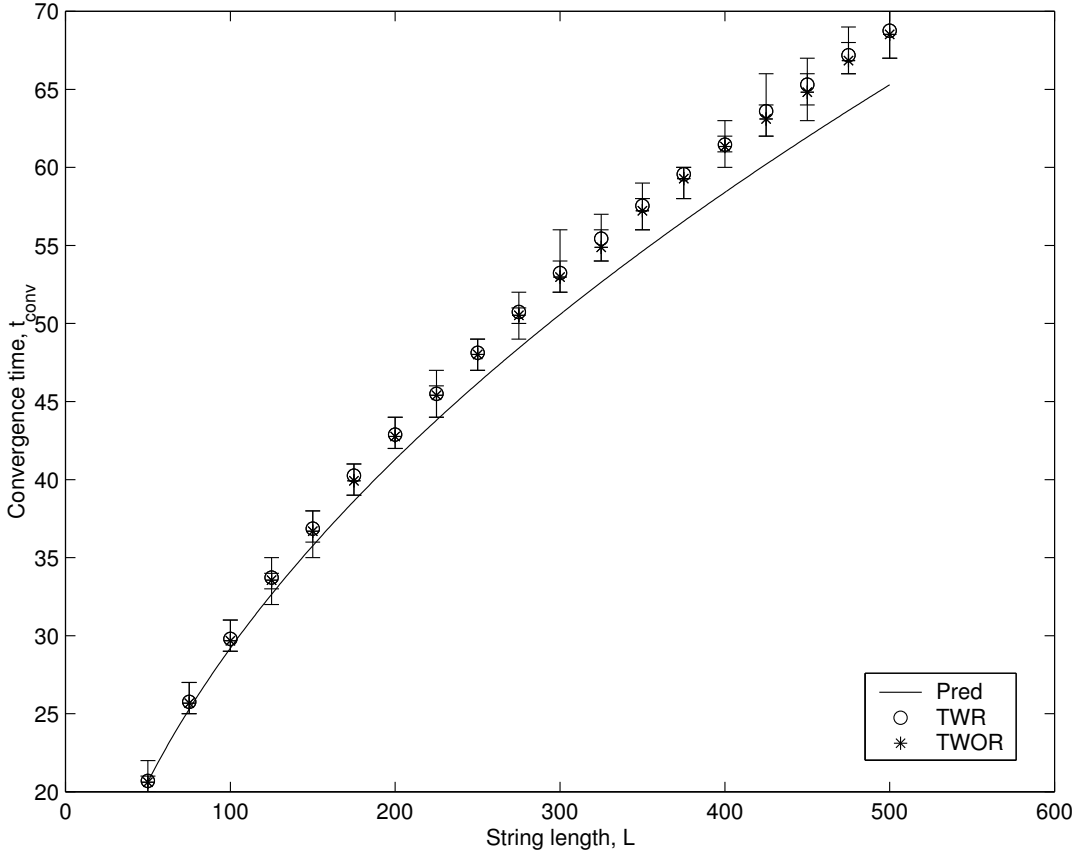
Experimental Methodology

- OneMax problem: Counting ones problem

$$f = \sum_{i=1}^{\ell} x_i$$

  - Linear, very easy to analyze
- Uniform crossover method
- Mutation is not used
- Results averaged over 100 independent runs

Convergence Time: TWR vs TWOR



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Population-Sizing Model

- Gambler's ruin model (Harik et al, 1997)
  - Combines BB supply & decision making model
  - Implicitly assumes TWOR

$$n = \frac{-2^{k-1}\log(\psi)\sqrt{\pi}}{d}\sqrt{\sigma_f^2 + \sigma_N^2}$$

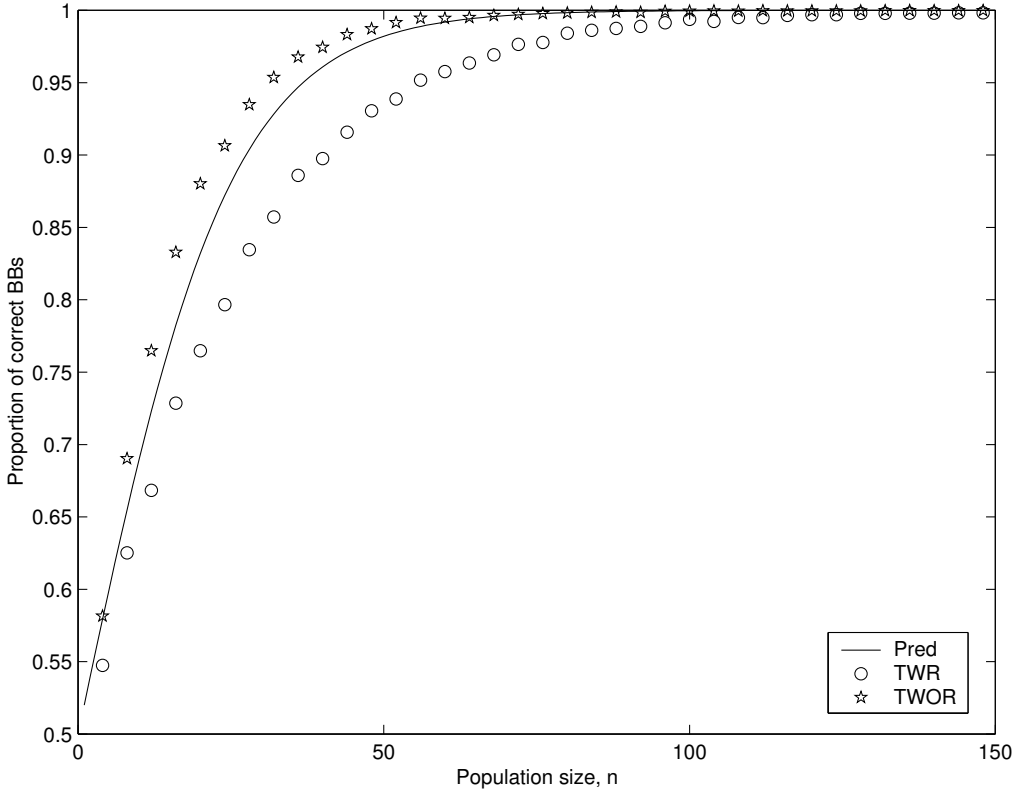
$k \rightarrow$ Building block size $\quad d \rightarrow$ Signal

$m \rightarrow$ No. of building blocks $\quad \psi \rightarrow$ Failure probability

$\sigma_f^2 \rightarrow$ Fitness variance $\quad \sigma_N^2 \rightarrow$ Noise variance

Population Size: TWR vs TWOR

No external noise, $\sigma_N^2 = 0$



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TWR vs TWOR

- Convergence time
  - TWR and TWOR have similar convergence times.
- Population size: Constant quality
  - Significant difference
  - TWR requires more population size
- Solution Quality: Constant population size
  - TWOR yields better solution quality

Apparent Added Noise

- TWR is a noisy selection scheme
  - Best individual gets s copies on average
  - It can also get n copies or 0 copies
  - TWOR: Best individual gets exactly s copies
- Incorporate noise as *apparent* external noise
  - Goldberg, Deb & Clark, 1992: Roulette wheel selection
- TWR = TWOR + External noise

No. Of Tournaments per Individual

- Individual in a tournament: Bernoulli
- Individual in n tournaments: Binomial
  - Success probability:

$$p = \frac{s}{n}$$

  - Average tournaments per individual: $\mu = s$
  - Variance in tournaments per individual:

$$\sigma^2 = s\frac{(n-s)}{n}$$

  - Small tournament sizes: $\sigma^2 \approx s$

Calculation Of Apparent Added Noise

- Selection scheme noise: $\sigma_s^2 = s$
- Units of squared individuals
- Need it in fitness squared terms

$$n = \frac{-2^{k-1}\log(\psi)\sqrt{\pi}}{d}\sqrt{\sigma_f^2 + \sigma_N^2}$$

- Extra tournament: $(f - \overline{f})$ change
- Fitness squared terms: $(f - \overline{f})^2$
- Average: $E\left[(f - \overline{f})^2\right] \propto \sigma_f^2$

Calculation Of Apparent Added Noise

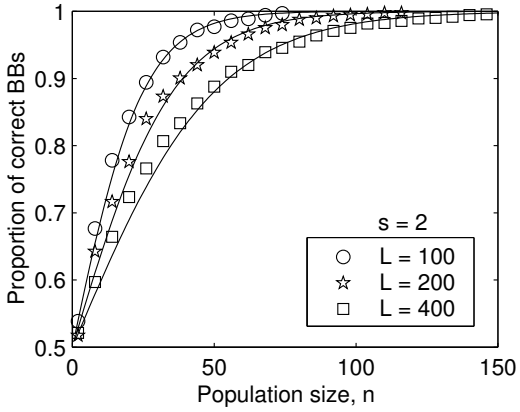
- (Fitness squared units)/(individual squared units):

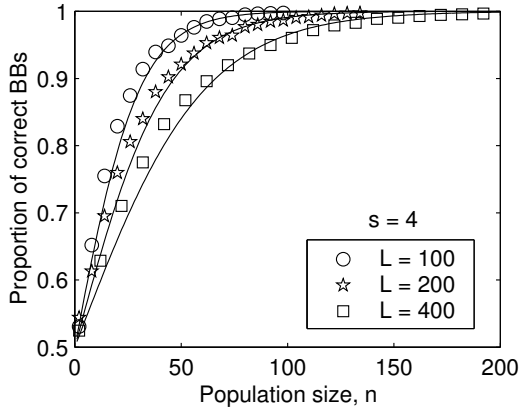
$$\propto \sigma_f^2$$

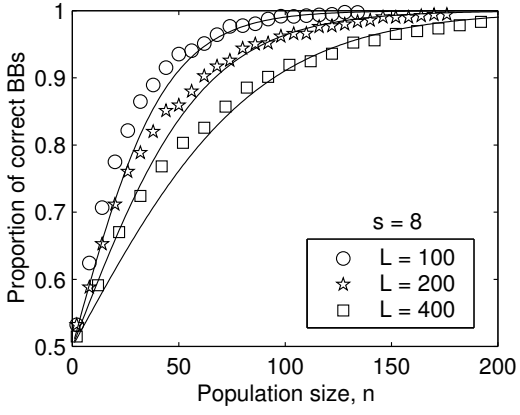
- Selection noise: $\sigma_s^2 \propto s\sigma_f^2$
- Apparent added noise: $\sigma_N^2 = c_o s \sigma_f^2$
- Population-sizing model for TWR:

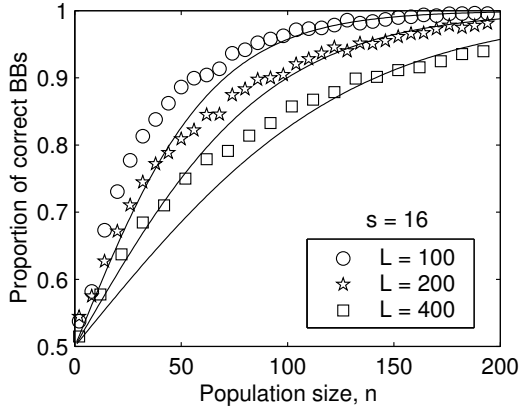
$$n = -2^{k-1} \log(\psi) \sqrt{\pi(1 + c_o s)} \frac{\sigma_f}{d}$$

Population-Sizing for TWR









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Summary

- Compared TWR and TWOR
  - Run durations are similar
  - TWR requires larger population
- Population-sizing model for TWR
  - Incorporates noise due to TWR
  - Accounted as apparent external noise
- Model useful for GA parameter setting
- Helpful in understanding GA mechanism

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