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Abstract

This paper analyzes the effects of tournament selection with replacement on the convergence time and population sizing for selectorecombinative genetic algorithms. This paper empirically demonstrates that the run duration remains the same and is not affected whether the tournament selection is performed with or without replacement. However, the population size required is more if tournament selection is performed with replacement rather than without replacement to attain the same level of accuracy. An approximate population sizing model is derived based on apparent added noise for the case of tournament selection with replacement. The proposed model is verified with experimental results.

1 Introduction

Tournament selection (Goldberg, Korb, & Deb, 1989) is one of the most widely used ordinal selection schemes. In tournament selection a specified number of individuals, s, is selected from the current population of size n. The best individual out of the s individuals gets a copy in the mating pool. The selection of s individuals can be performed either *with replacement* or *without replacement*. In selection with replacement the individuals selected for the current tournament are candidates for other tournaments. On the other hand, if selected without replacement the individuals once selected are not candidates for other tournaments.

In genetic algorithms (GAs) literature, tournament selection without replacement (TSWOR) has received considerable analytical attention. Both the population size required at different tournament size values and the convergence time with and without noisy function evaluation have been successfully considered. On the other hand, tournament selection with replacement (TSWR) has not received separate scrutiny, and is usually considered to be equivalent to TSWOR. In this paper we empirically demonstrate that TSWR requires more number of function evaluations for attaining the same accuracy as TSWOR. We demonstrate that even though the run duration is not affected, the population size required for successful convergence depends on whether we select with or without replacement. In this paper we propose to model this discrepancy as an apparent noise similar to that proposed by Goldberg, Deb, and Clark (1992) for the case of roulette-wheel selection. We also develop an approximate model based on the apparent added noise to adjust the population size required for selectorecombinative GAs that employ TSWR.

This paper is structured as follows: first a brief literature review on selection schemes is presented. Subsequently, we compare the convergence time of GAs with TSWR to that of TSWOR. We then proceed to address the issue of population sizing required for optimal convergence for the case of TSWR.

2 Literature Review

A complete exhaustive literature review on selection schemes employed in GAs is beyond the scope of this paper and hence a very brief overview is presented. The various selection schemes that exist in GA literature can be broadly classified into two classes: (1) proportionate schemes, and (2) ordinal-based schemes. Proportionate schemes (Holland, 1975; Brindle, 1981; Booker, 1982; Baker, 1987; Grefenstette & Baker, 1989) select an individual based on its relative fitness value compared to others. The selection pressure in these procedures are dependent on the fitness distribution of the population. Ordinal-based schemes (Brindle, 1981; Schwefel, 1981; Baker, 1985; Baker, 1987; Grefenstette & Baker, 1989; Goldberg, 1989b; Goldberg, Korb, & Deb, 1989; Muhlenbein & Voosen, 1993) select an individuals based on its ranking in the population, where the individuals are ranked based on their fitness values. The selection pressure of ordinal schemes is independent of fitness distribution of the population. Unlike proportionate schemes ordinal schemes do not suffer from scaling problems (Goldberg, 1989a; Whitley, 1989) and are therefore preferred.

3 Convergence Time

Convergence models for different selection schemes were presented by Goldberg and Deb (1991). They modeled the change in proportion of the individual strings in the population, when using only selection. Muhlenbein and Voosen (1993) employed the concept of *selection intensity* (Bulmer, 1980) for convergence analysis of selection schemes in GAs. Thierens and Goldberg (1994) developed convergence models for several selection schemes using normal distribution analysis for OneMax problem. Back (1995) and Miller and Goldberg (1996b) extended the tournament selection model to handle tournament sizes larger than two. Both employed order statistics in their model. Miller and Goldberg (1996a) refined the convergence model to handle external noise. In this study external noise is considered to be zero and hence the convergence model by Muhlenbein and Voosen (1993) can be employed and is given below

$$t_{\text{conv}} = \frac{\pi\sqrt{\ell}}{2I}, \tag{1}$$

where, t_{conv} is the convergence time, ℓ is the string length, and I is the selection intensity, which measures the magnitude of selection pressure provided for a selection scheme. Selection intensity for various values of tournament sizes, s, is tabulated elsewhere (Miller & Goldberg, 1996b; Miller & Goldberg, 1996a).

The convergence model (eqn. 1) is compared with experimental results in fig. 1 for both tournament selection with and without replacement as a function of string length. The experiment is for OneMax problem with uniform crossover, crossover probability of 1.0 and the probability of swapping alleles 0.5. The tournament size considered is 2, and the population size 1000. This population size is very high and is so chosen as to avoid population sizing effects. The divergence between the convergence time model and empirical results are due to hitch-hiking and other studies have shown better agreement with increasing mixing of building blocks (Thierens, 1995).

It can be easily seen that both tournament selection with and without replacement have similar convergence time. This similarity in run duration between TSWR and TSWOR exists even in the

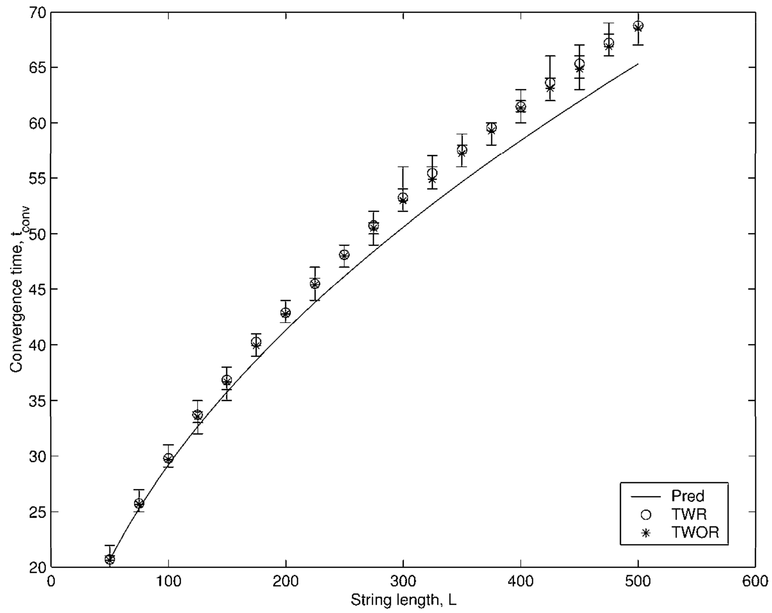

Figure 1: Convergence time as predicted by eqn. (1) verified with empirical results. The experimental results are that of a OneMax problem of different sizes for TSWR and TSWOR. Both TSWR and TSWOR have similar convergence time. The empirical results are averaged over 100 runs. The agreement between the theoretical and empirical result can be improved by increasing mixing of building blocks.

case of small population sizes. This indicates that the duration is not affected on whether we select with replacement or not. The similarity in convergence time between TSWR and TSWOR can be justified by the fact that in both TSWR and TSWOR, the best individual among selected s individuals is always the winner. The only difference is that the quality of converged solution differs between GAs that use TSWR and TSWOR. The similarity between TSWR and TSWOR can also be justified by the fact that on an average both with and without replacement allocate s copies to the best individual and zero copies to the worst individual in the population.

In this section we demonstrated the similarity in convergence time between TSWR and TSWOR, and the next logical step is to address the issue of population sizing for optimal convergence. This topic is considered next, where we empirically show that the difference in population sizing between TSWR and TSWOR is significant and propose a method for modeling population sizing for TSWR.

4 Population Sizing

Population size is a very important factor in determining the solution quality obtained through a GA run. Goldberg, Deb, and Clark (1992) proposed population sizing models for different selection schemes. Their model is based on deciding correctly between the best BB in a partition and the second best BB in the same partition. They incorporated the presence of noise arising from other partitions into their model. However, they assumed that if wrong BBs were chosen in the first generation, the GAs would be unable to recover from the error. Harik, Cantu-Paz, Goldberg, and Miller (1997) extended the above model by incorporating cumulative effects of decision making over time rather than in the first generation only. They modeled the decision making between the best and second best BB in a partition as a gambler's ruin problem. This model is based

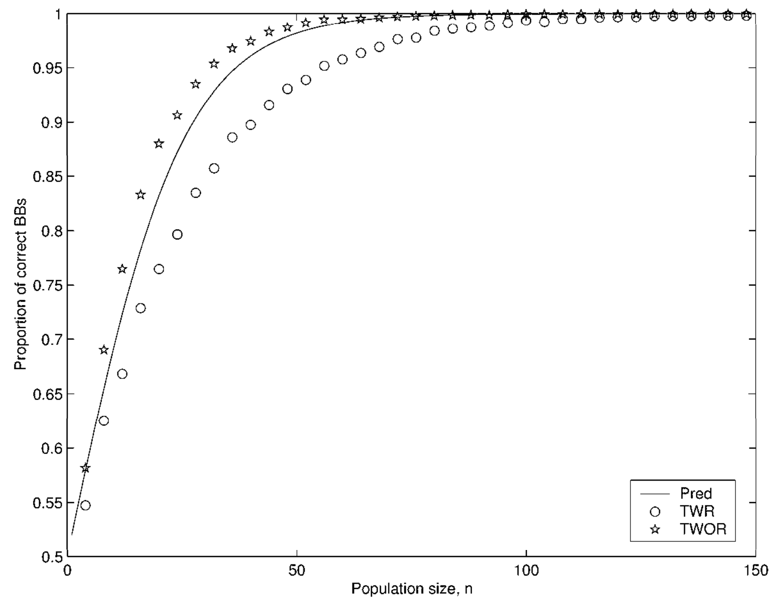

Figure 2: Proportion of correct BBs predicted by the population sizing model (eqn. 2) without the noise term ($\sigma_N^2 = 0$) for a 200-bit OneMax problem for tournament selection with (TSWR) and without replacement (TSWOR). The results indicate that TSWOR agrees with the population sizing model and TSWR does not. TSWR requires more population size than predicted by eqn. (2) to attain the same accuracy. The empirical results are averaged over 100 runs.

on the assumption that the selection process used is tournament selection without replacement. Miller (1997) refined this model to predict population sizing in the presence of external noise. The population sizing model derived by Miller (1997) is proposed below:

$$n = -\frac{-2^{k-1}\log(\psi)}{d}\sqrt{\pi\left(\sigma_F^2 + \sigma_N^2\right)} \tag{2}$$

where, k is the number of building blocks (BBs) in the problem, ψ is the failure rate, d is the difference in fitness between the best and the second best BB, σ_F^2 is the fitness variance, and σ_N^2 is the noise variance.

Our first step is to verify the above population sizing equation with computational results for both TSWR and TSWOR. Figure 2 depicts the success rate $1 - \psi$ as a function of population size for a 200-bit OneMax problem. Since the fitness function is not noisy, the external noise variance σ_N^2 is set to zero. It can be seen that the population sizing model does not agree with computational results for both TSWR and TSWOR for small population sizes. This is due to the reason that eqn. (2) is an approximation and is known to estimate the success rate conservatively for small population sizes. The agreement of the analytical and experimental results for the case of TSWR can be increased by using a more accurate form of the above model (Miller, 1997). From fig. 2 we can easily see that though the population sizing model agrees for the case of TSWOR, the same is not true for TSWR. The results for TSWR indicate that the population size required to reach the same level of accuracy is more than that predicted by the model. This discrepancy can be explained, because the population sizing model without the noise term does not account for the noise of selection, and TSWR is certainly a noisy scheme when compared to TSWOR. In TSWR the best individual in the population on an average has s copies in the mating pool. It can also have all n copies or none at all, where as in TSWOR the best individual has *exactly* s copies in the

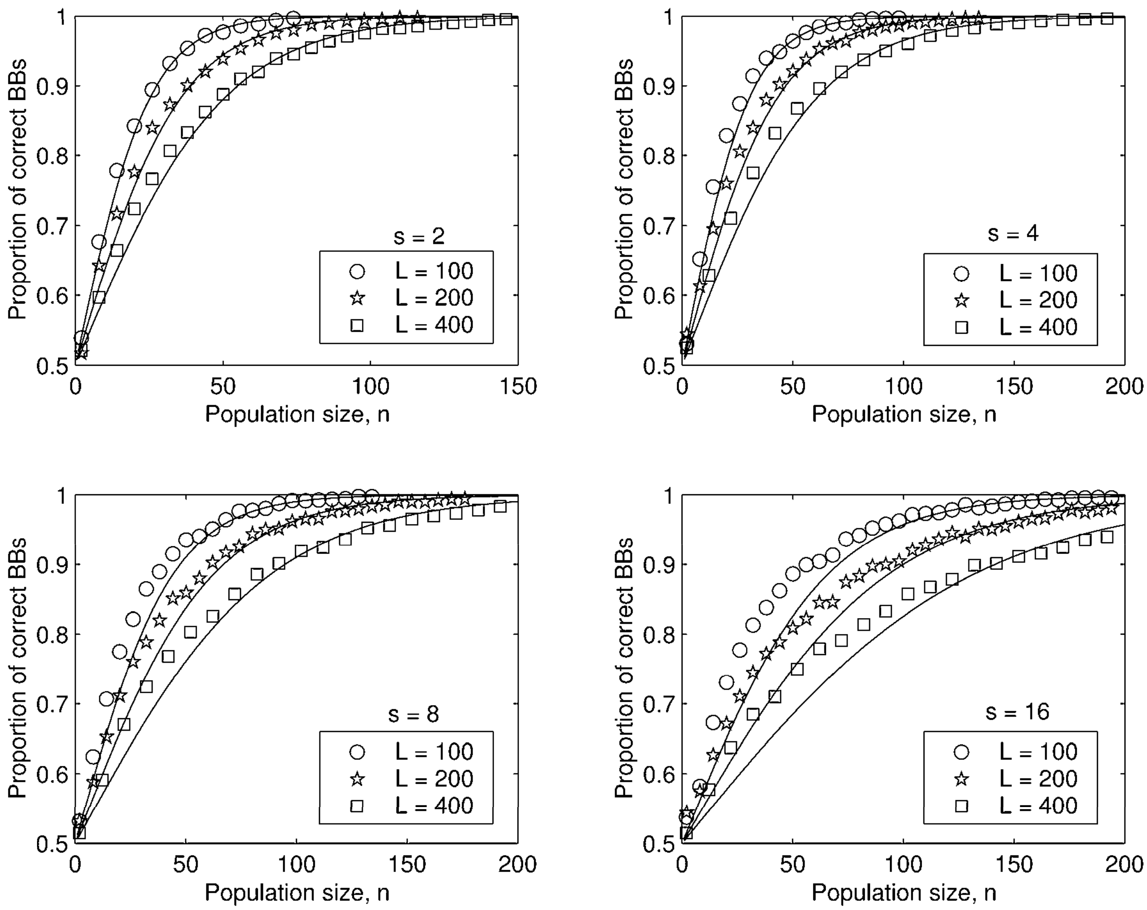

Figure 3: Proportion of correct BBs, $1 - \psi$, predicted by the population sizing model (eqn. 6) as a function of population size n at different tournament size values and for different problem sizes compared to computational results. The computational results are for the OneMax problem with tournament selection with replacement (TSWR). The results are averaged over 100 runs. For all cases $c_o = 0.25$.

mating pool.

Therefore, we propose to account for the noise due to TSWR as an apparent external noise in the population sizing model (eqn. 2). This procedure is similar to that proposed by Goldberg, Deb, and Clark (1992) for roulette-wheel selection. To quantify the noise due to selection, recognize that the process of selecting an individual is a Bernoulli trial and this process is repeated n/s times. Therefore the process of selecting an individual i in n/s trials is binomially distributed with probability p_i. For an s-wise tournament, the probability of individual i being in s unique individuals is given by

$$p_i = \frac{\binom{n-1}{s-1}}{\binom{n}{s}} = \frac{s}{n}. \tag{3}$$

The mean number of tournaments that the individual i participates in is given by $(n/s)p_i$ and the variance is given by $(n/s)p_i\,(1-p_i)$. Summing over all individuals we get a variance

$$\frac{s}{n}\sum_{i=1}^{n/s}\frac{n}{s}p_i\,(1-p_i)=\frac{n-s}{n}\approx 1. \tag{4}$$

Since this process is repeated s times to maintain constant population size n and the mean number of tournaments that an individual i participates in is given by $s(n/s)p_i = s$. The variance of tournaments that an individual i participates in is given by $s(n-s)/n \approx s$. This variance due to the noise of TSWR is in the units of squared individuals. To use this variance in eqn. (2), we have to convert it into units of squared fitness. To convert to units of squared fitness, we recognize that an individual must change by an amount equal to some proportion of the population fitness variance to increase or decrease its numbers by s. Therefore, the variance due to TSWR in fitness terms is the product of variance in terms of number and some proportion of the fitness variance. Thus, the appropriate variance due to TSWR, σ_s^2, can be written as,

$$\sigma_s^2 = c_o s\sigma_F^2, \tag{5}$$

where c_o is a proportionality constant and is empirically determined. A wide range of tournament size values $2 \leq s \leq 20$ have been used to determine the value of c_o and in all cases c_o is evaluated to be 0.25. The fact that c_o is constant over a wide range of s values justifies the claim that the noise due to selection is proportional to the fitness variance.

Using this noise due to selection as an external noise term in eqn. (2), i.e., $\sigma_N^2 = \sigma_s^2$, the approximate population sizing model for TSWR can be written as

$$n = -\frac{-2^{k-1}\log(\psi)}{d}\sqrt{\pi\,(1+c_o s)\,\sigma_F^2} \tag{6}$$

Using the above equation, the success rate $1-\psi$ is computed for different tournament size values and different problems sizes as a function of population size. The results are verified with computational results in 3. The experimental results are for the case of the OneMax problem with string lengths, ℓ = 100, 200, and 300, and for tournament sizes s = 2, 4, 8, and 16. The empirical results are averaged over 100 independent runs. The results show that the agreement between the analytical and empirical relations increases significantly with the addition of the apparent noise term.

5 Conclusions

In this study tournament selection with replacement (TSWR) has been considered and its effects on convergence time and population sizing of a selectorecombinative GAs investigated. The analysis yielded some interesting conclusions: (1) The run duration is the same for both TSWR and tournament selection without replacement (TSWOR). (2) The population size required by TSWR is greater than that required by TSWOR. This paper has derived and verified an approximate model for the population sizing of TSWR based on an apparent added noise to the population sizing model of TSWOR. This model may be used for GA run parameter setting, competent GA design, or simply to advance our understanding of GA mechanism.

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